NAME _____

AP Calculus AB Summer Math Packet

DIRECTIONS/INFORMATION:

- This packet contains review problems from your most recent math class and represents the types of mathematics knowledge your teacher expects you to have **before** entering AP Calculus AB.

- The packet is divided into eight one-week sections that will allow you to develop a schedule for completing the entire packet. Follow the directions given in each section of the packet. Show your work and give full explanations where necessary. If additional space is needed, complete the work on a separate sheet of paper and attach it to the packet.

- **Watch the videos in the links below BEFORE attempting the problems.**

Week 1 Videos:
- https://www.youtube.com/watch?v=ZFPkQkURSxk
- https://www.youtube.com/watch?v=oORnGaJp1pk

Week 2 Videos:
- https://www.youtube.com/watch?v=djT6-YamHaA
- https://www.youtube.com/watch?v=hZpc1rNZjlY

Week 3 Video:
- https://www.youtube.com/watch?v=zqCokP2vDGo

Week 4 Videos:
- https://www.youtube.com/watch?v=cjXseEPP9vc
- https://www.youtube.com/watch?v=kvhpu1TkSjI

Week 5 Video:
- https://www.youtube.com/watch?v=Rf05H8ogHLg

Week 6 Videos:
- https://www.youtube.com/watch?v=VMAMARmmDac
- https://www.youtube.com/watch?v=kEcbxiLeGTc

Week 7 Videos:
- https://www.youtube.com/watch?v=AyJsHC4OYCM
- https://www.youtube.com/watch?v=fOrOeZA-vdY
- https://www.youtube.com/watch?v=HbtuSC_WOW0
- https://www.youtube.com/watch?v=UkjgJQaGx98

Week 8 Videos:
- https://www.youtube.com/watch?v=cyn6CDnAlcA
- https://www.youtube.com/watch?v=9Yz-RCdS2Tg
- https://www.youtube.com/watch?v=5NyeGzbBJQM
- https://www.youtube.com/watch?v=UwFlrPNf5ZE

Thank you in advance for completing this packet by the first day of school. We look forward to working with you.

Week 1 – Function Notation

- If $f(x)=4x-x^2$, find

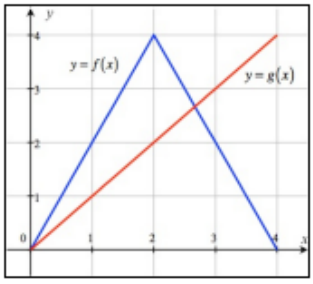
1. $f(4)-f(-4)$
2. $\sqrt{f\left(\frac{3}{2}\right)}$
3. $\frac{f(x+h)-f(x-h)}{2h}$

- If $V(r)=\frac{4}{3}\pi r^3$, find

4. $V\left(\frac{3}{4}\right)$
5. $V(r+1)-V(r-1)$
6. $\frac{V(2r)}{V(r)}$

- If $f(x)$ and $g(x)$ are given in the graph below, find

7. $(f-g)(3)$
8. $f(g(3))$



- If $f(x)=x^2-5x+3$ and $g(x)=1-2x$, find

9. $f(g(x))$

- If $f(x)=\begin{cases}\sqrt{x+2}-2, & x\geq 2\\ x^2-1, & 0\leq x<2\\ -x, & x<0\end{cases}$, find

10. $f(0)-f(2)$
11. $\sqrt{5-f(-4)}$
12. $f(f(3))$

Week 2 – Domain & Range

• Find the domain of the following functions using interval notation:

1. $f(x)=3$

2. $y=x^3-x^2+x$

3. $y=\dfrac{x^3-x^2+x}{x}$

4. $y=\dfrac{x-4}{x^2-16}$

5. $f(x)=\dfrac{1}{4x^2-4x-3}$

6. $y=\sqrt{2x-9}$

7. $f(t)=\sqrt{t^3+1}$

8. $f(x)=\sqrt[5]{x^2-x-2}$

9. $y=5^{x^2-4x-2}$

10. $y=\log(x-10)$

11. $y=\dfrac{\sqrt{2x+14}}{x^2-49}$

12. $y=\dfrac{\sqrt{5-x}}{\log x}$

Find the range of the following functions.

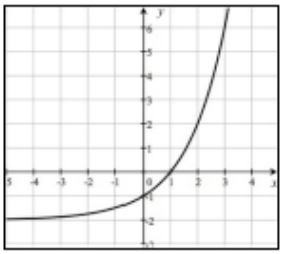
13. $y=x^4+x^2-1$

14. $y=100^x$

15. $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

16.



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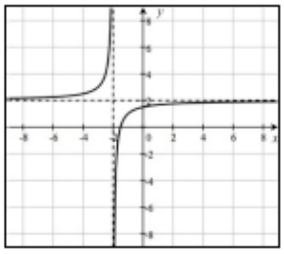

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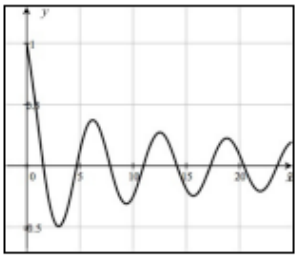
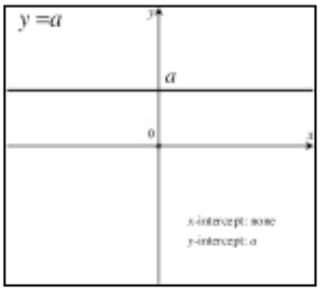
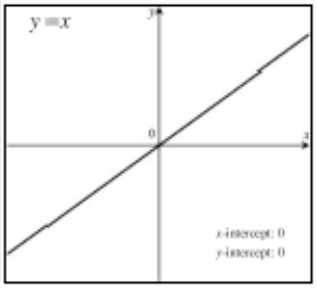


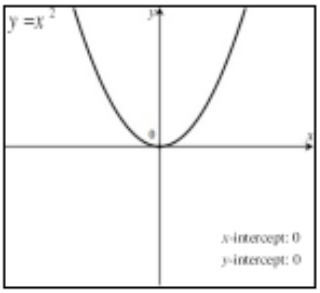
There are certain graphs that occur all the time in calculus and students should know the general shape of them, where they hit the x-axis (zeros) and y-axis (y-intercept), as well as the domain and range. There are no assignment problems for this section other than students memorizing the shape of all of these functions. In section 5, we will talk about transforming these graphs.



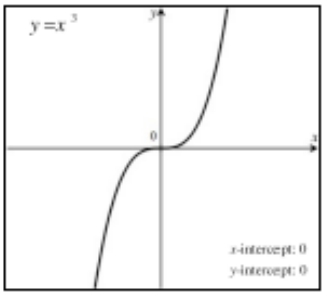
Function: $y = a$
Domain: $(-\infty, \infty)$
Range: $[a, a]$



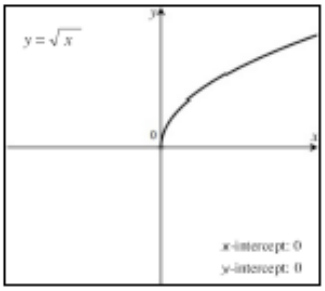
Function: $y = x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



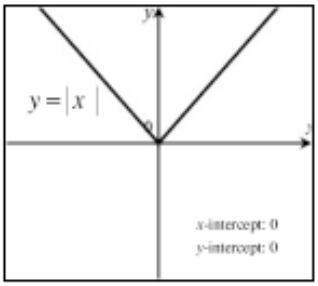
Function: $y = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$



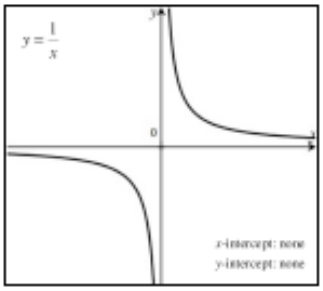
Function: $y = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



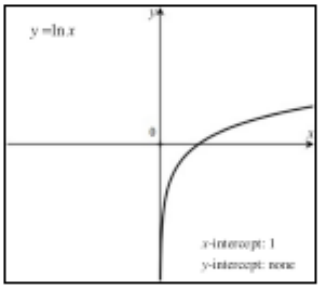
Function: $y = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$



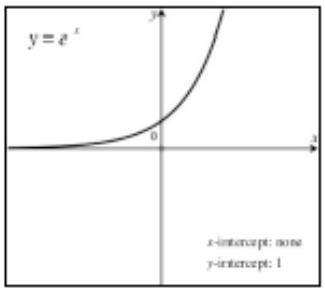
Function: $y = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$



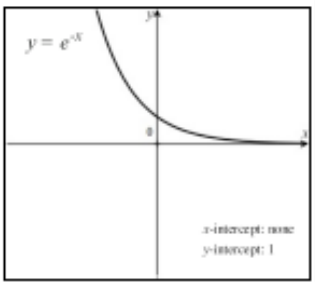
Function: $y = \dfrac{1}{x}$
Domain: $x \neq 0$
Range: $y \neq 0$



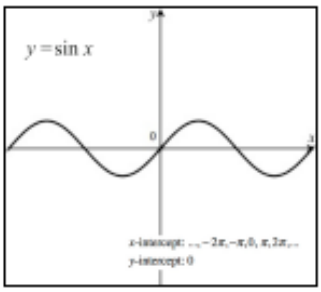
Function: $y = \ln x$
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$



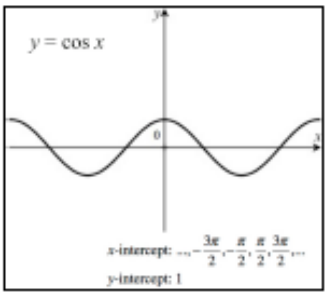
Function: $y = e^x$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$



Function: $y = e^{-x}$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$



Function: $y = \sin x$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$



Function: $y = \cos x$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

Week 3 – Linear Functions

1. Find the equation of the line in point-slope form, with the given slope, passing through the given point.
   a. $m=-7, (-3,-7)$
   b. $m=\frac{-1}{2}, (2,-8)$
   c. $m=\frac{2}{3}, \left(-6,\frac{1}{3}\right)$

2. Find the equation of the line in point-slope form, passing through the following points.
   a. $(-3,6)$ and $(-1,2)$
   b. $(-7,1)$ and $(3,-4)$
   c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.
   a. $(5,-3)$, $x+y=4$
   b. $(-6,2)$, $5x+2y=7$
   c. $(-3,-4)$, $y=-2$

4. Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

5. Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

Week 4 – Asymptotes

• Find any vertical and horizontal asymptotes and if present, the location of holes, for the graph of

1. $y=\frac{x-1}{x+5}$

2. $y=\frac{8}{x^2}$

3. $y=\frac{2x+16}{x+8}$

4. $y=\frac{2x^2+6x}{x^2+5x+6}$

5. $y=\frac{x}{x^2-25}$

6. $y=\frac{x^2-5}{2x^2-12}$

7. $y=\frac{4+3x-x^2}{3x^2}$

8. $y=\frac{5x+1}{x^2-x-1}$

9. $y=\frac{1-x-5x^2}{x^2+x+1}$

10. $y=\frac{x^3}{x^2+4}$

11. $y=\frac{x^3+4x}{x^3-2x^2+4x-8}$

12. $y=\frac{10x+20}{x^3-2x^2-4x+8}$

13. $y=\frac{1}{x}-\frac{x}{x+2}$ (hint: express with a common denominator)

Week 5 –Trig Identities

Fundamental Trig Identities

$\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\cot x = \frac{1}{\tan x}$, $\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$

Sum Identities

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Double Angle Identities

$\sin(2x) = 2\sin x \cos x$ $\quad$ $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

• Verify the following identities.

1. $(1+\sin x)(1-\sin x) = \cos^2 x$

2. $\sec^2 x + 3 = \tan^2 x + 4$

3. $\frac{1-\sec x}{1-\cos x} = -\sec x$

4. $\frac{1}{1+\tan x} + \frac{1}{1+\cot x} = 1$

5. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

6. $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

7. $\csc 2x = \frac{\csc x}{2\cos x}$

8. $\frac{\cos 3x}{\cos x} = 1 - 4\sin^2 x$

Week 6 – Trig Equations

• Solve for x on $[0,2\pi)$

1. $\sin^2 x = \sin x$

2. $3\tan^3 x = \tan x$

3. $\sin^2 x = 3\cos^2 x$

4. $\cos x + \sin x \tan x = 2$

5. $\sin x = \cos x$

6. $2\cos^2 x + \sin x - 1 = 0$

7. Solve for x on $[0,2\pi)$: $\dfrac{x-\pi}{\cos^2 x} < 0$

Week 7 – Limits

1. Evaluate the following

a. $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}=$

b. $\lim_{x\to+\infty}\frac{7x^4+4x^3}{8x^4+4x^2-3}$

c. $\lim_{x\to 7}\frac{x^2-49}{x-7}=$

d. $\lim_{x\to 0}\frac{\sqrt{5x+16}-4}{-7x}=$

e. $\lim_{x\to 1}\frac{1-x}{\sqrt{12-3x}-3}=$

f. $\lim_{x\to 1}\frac{x^3-1}{x-1}=$

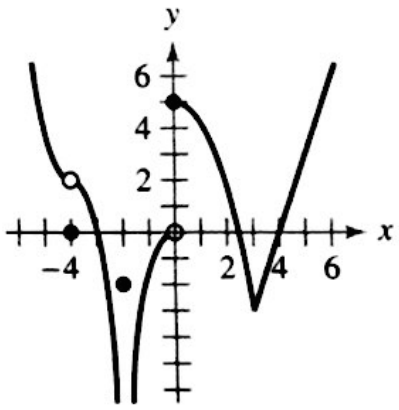
g. $\lim_{x\to 0}\frac{\sin 5x}{x}=$

h. $\lim_{x\to 0}\frac{1-\cos x}{\sin x}=$

i. $\lim_{x\to 0}\frac{1-\cos x}{1+\sin x}=$

j. $\lim_{x\to 0}\frac{\sin x}{x^2+2x}=$

2. For the function f whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.



(a) $\lim_{x\to -4} f(x) =$

(b) $f(-4) =$

(c) $\lim_{x\to -2} f(x) =$

(d) $\lim_{x\to 0^-} f(x) =$

(e) $\lim_{x\to 0^+} f(x) =$

(f) $\lim_{x\to 0} f(x) =$

(g) $\lim_{x\to 3} f(x) =$

(h) $\lim_{x\to 4} f(x) =$

Week 8 – Derivatives

Find the derivative, $\frac{dy}{dx}$, of each function.

(HINT: #8, 9 rewrite first by dividing the terms in the numerator by x^2 or x)

1. $y = 8$

2. $y = x^6$

3. $y = \frac{1}{x^7}$

4. $y = \sqrt[5]{x}$

5. $y = -2x^2 + 3x - 6$

6. $y = x^2 + 5 - \frac{3}{x^2}$

7. $y = \sqrt{x} - 6\sqrt[3]{x}$

8. $y = \frac{x^3 - 3x^2 + 4}{x^2}$

9. $y = \frac{2x^2 - \sqrt{x^3} + 1}{x}$

Find an equation of the tangent line to the graph of f at the given point.

10. $f(x) = x^4 - 3x^2 + 2$, Point $(1, 0)$

11. $f(x) = \frac{2}{\sqrt[4]{x^3}}$, Point $(1, 2)$

Find the derivative of each function.

12. $y = \frac{1}{x} - 3\sin x$

13. $y = \frac{1}{8x^3} + 2\cos x$

14. $y = \frac{2}{\sqrt[3]{x}} + 3\cos x$